OCR
Oxford Cambridge and RSA

Tuesday 21 June 2016 – Morning

A2 GCE MATHEMATICS

4735/01 Probability & Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4735/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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Answer **all** the questions.

1 Ten archers shot at targets with two types of bow. Their scores out of 100 are shown in the table.

Archer	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Bow type *P*	95	97	92	85	87	92	90	89	98	77
Bow type *Q*	91	91	88	90	80	88	93	85	94	84

(i) Use the sign test, at the 5% level of significance, to test the hypothesis that bow type P is better than bow type Q. **[7]**

(ii) Why would a Wilcoxon signed rank test, if valid, be a better test than the sign test? **[1]**

2 Low density lipoprotein (LDL) cholesterol is known as 'bad' cholesterol.
15 randomly chosen patients, each with an LDL level of 190 mg per decilitre of blood, were given one of two treatments, chosen at random. After twelve weeks their LDL levels, in mg per decilitre, were as follows.

Treatment *A*	189	168	176	186	183	187	188	
Treatment *B*	177	179	173	180	178	170	175	174

Use a Wilcoxon rank sum test, at the 5% level of significance, to test whether the LDL levels of patients given treatment B are lower than the LDL levels of patients given treatment A. **[8]**

3 The table shows the joint probability distribution of two random variables X and Y.

		Y		
		0	1	2
X	0	0.07	0.07	0.16
	1	0.06	0.09	0.15
	2	0.07	0.14	0.19

(i) Find Cov(X, Y). **[5]**

(ii) Are X and Y independent? Give a reason for your answer. **[2]**

(iii) Find $\mathrm{P}(X = 1 \mid XY = 2)$. **[2]**

4 The continuous random variable Y has a uniform (rectangular) distribution on $[a, b]$, where a and b are constants.

(i) Show that the moment generating function $\mathrm{M}_Y(t)$ of Y is $\dfrac{(\mathrm{e}^{bt} - \mathrm{e}^{at})}{t(b-a)}$. **[2]**

(ii) Use the series expansion of e^x to show that the mean and variance of Y are $\frac{1}{2}(a+b)$ and $\frac{1}{12}(b-a)^2$, respectively. **[7]**

5 Events A and B are such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \mid B') = 0.75$.

(i) Find $\mathrm{P}(A \cap B)$ and $\mathrm{P}(A \cup B)$. **[6]**

(ii) Determine, giving a reason in each case,
(a) whether A and B are mutually exclusive,
(b) whether A and B are independent. **[2]**

(iii) A further event C is such that $\mathrm{P}(A \cup B \cup C) = 1$ and $\mathrm{P}(A \cap B \cap C) = 0.05$. It is also given that $\mathrm{P}(A \cap B' \cap C) = \mathrm{P}(A' \cap B \cap C) = x$ and $\mathrm{P}(A \cap B' \cap C') = 2x$.
Find $\mathrm{P}(C)$. **[3]**

6 Andrew has five coins. Three of them are unbiased. The other two are biased such that the probability of obtaining a head when one of them is tossed is $\frac{3}{5}$.

Andrew tosses all five coins. It is given that the probability generating function of X, the number of heads obtained on the unbiased coins, is $\mathrm{G}_X(t)$, where

$$\mathrm{G}_X(t) = \tfrac{1}{8} + \tfrac{3}{8}t + \tfrac{3}{8}t^2 + \tfrac{1}{8}t^3.$$

(i) Find $G_Y(\mathrm{t})$, the probability generating function of Y, the number of heads on the biased coins. **[3]**

(ii) The random variable Z is the total number of heads obtained when Andrew tosses all five coins. Find the probability generating function of Z, giving your answer as a polynomial. **[3]**

(iii) Find $\mathrm{E}(Z)$ and $\mathrm{Var}(Z)$. **[6]**

(iv) Write down the value of $\mathrm{P}(Z = 3)$. **[1]**

7 A continuous random variable Y has cumulative distribution function

$$\mathrm{F}(y) = \begin{cases} 0 & y < a \\ 1 - \dfrac{a^5}{y^5} & y \geqslant a \end{cases}$$

where a is a parameter.

Two independent observations of Y are denoted by Y_1 and Y_2. The smaller of them is denoted by S.

(i) Show that $P(S > \mathrm{s}) = \dfrac{a^{10}}{s^{10}}$ and hence find the probability density function of S. **[5]**

(ii) Show that S is not an unbiased estimator of a, and construct an unbiased estimator of a, T_1 based on S. **[4]**

(iii) Construct another unbiased estimator of a, T_2, of the form $k(Y_1 + Y_2)$, where k is a constant to be found. **[4]**

(iv) Without further calculation, explain how you would decide which of T_1 and T_2 is the more efficient estimator. **[1]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		H_0: $p = ½$, H_1: $p > ½$ Find signs of differences Obtain 7+, 3- Attempt $P(X \geq 7)$ or $P(X \leq 3)$ 0.1719 "0.1719" > 0.05, so do not reject H_0 Insuff. evidence that type P is better.	B1 M1 A1 M1ft A1ft M1 A1 **[7]**	For both. Allow any sensible hypotheses. +++-++-++- or vv or vv Allow 0.172 (0.0547 from 8+) Ft candidate's *p*. In context, not over-assertive. Cwo.	Attempt to find CR. M1 (not ft) $X \geq 9$ or $X \leq 1$ A1 (not ft) "7" (or "3") not in CR, so d.n.r. H_0 ft NOT "suff evidence that there is no difference between the bows."
	(ii)		Magnitude of differences taken into account.	B1 **[1]**	Uses more information. More powerful.	
2			H_0: $m_A = m_B$, H_1: $m_B < m_A$ Attempt ranks 15, 1, 6, 12, 11, 13, 14; 7, 9, 3, 10, 8, 2, 5, 4 $R_m = 72$ $W = 40$ CV = 41 "40" < 41 reject H_0 Evidence that treatment *B* is more effective.	B1 M1 A1 A1 A1 B1 M1 A1 **[8]**	For both. Allow any sensible hypotheses. Ft TS and CV. In context, not over-assertive. Cwo.	
3	(i)		$E(X) = 1.1$ $E(Y) = 1.3$ $E(XY) = 1.43$ $Cov(XY) =$ "1.43" – "1.1"×"1.3" 0	B1 B1 B1 M1 A1 **[5]**		
	(ii)		e.g. $P(X = 0) \times P(Y = 1) \neq P(X = 0, Y = 1)$ Not independent.	M1 A1 **[2]**	Or conditional probs. Consider any of (0,y),(2,y)	
	(iii)		0.15/(0.15 + 0.14) 0.517	M1 A1 **[2]**	Allow $\frac{15}{29}$	

Question			Answer	Marks	Guidance	
4	(i)		$\int_a^b \frac{1}{b-a}e^{tx}dx$	M1		
			$\frac{e^{bt}-e^{at}}{t(b-a)}$ AG	A1	Need $\left[\frac{e^{tx}}{t(b-a)}\right]_a^b$	
				[2]		
	(ii)		$\frac{(1+bt+\frac{b^2t^2}{2}+\frac{b^3t^3}{6}+(...))-(1+at+\frac{a^2t^2}{2}+\frac{a^3t^3}{6}+(...))}{t(b-a)}$	M1	As far as terms in t^2. Allow num only or sign error. Use of $M''(0)-(M'(0))^2$ M1A1 as main scheme. [½(b^2-a^2)+⅓t(b^3-a^3)]/(b-a) A1 E(Y)= $\frac{b+a}{2}$ A1	.
			$1+\frac{(b^2-a^2)}{(b-a)}\frac{t}{2}+\frac{(b^3-a^3)}{(b-a)}\frac{t^2}{6}$ allow(b-a)/(b-a)for 1	A1	$\frac{b^3-a^3}{3(b-a)}$ A1	
			Simplify 3rd term to $\frac{1}{6}(b^2+ab+a^2)t^2$	A1	$\frac{b^3-a^3}{3(b-a)}-\frac{(b+a)^2}{4}$ M1	
			$\mathrm{E}(Y)=\frac{b+a}{2}$ AG	A1	$\frac{(b-a)^2}{12}$ A1 CWO	
			$[\mathrm{E}(Y^2)]=\frac{b^2+ab+a^2}{3}$ oe	A1		
			Use $\mathrm{Var}(Y)=\mathrm{E}(Y^2)-(\mathrm{E}(Y))^2$	M1		
			$\frac{(b-a)^2}{12}$ AG CWO	A1		
				[7]		

Question			Answer	Marks	Guidance	
5	(i)		$P(A\cap B')=0.75\times0.4=0.3$ $P(A\cap B)=0.5-"0.3"=0.2$ $P(A\cup B)=0.5+0.6-"0.2"=0.9$	M1A1 M1A1 M1A1 **[6]**		
	(ii)	(a) (b)	No, $P(A\cap B)\neq 0$ oe No, $0.5\times0.6\neq0.2$ oe	B1 B1 **[2]**		
	(iii)		$P(A'\cap B'\cap C)=0.1$ soi $x=0.1$ $P(C)=2x+0.05+0.1=0.35$	B1ft B1 B1 **[3]**	1 – (i)	
6	(i)		P(0) = 0.16, P(1) = 0.48, P(2) = 0.36 $G_Y(t)=0.16+0.48t+0.36t^2$	B1 M1A1 **[3]**		
	(ii)		$G_X(t)\times G_Y(t)$ soi $0.02+0.12t+0.285t^2+0.335t^3+0.195t^4+0.045t^5$	M1 A1A1 **[3]**	At least 4 terms correct; All correct.	
	(iii)		$E(Z)=G_Z'(1)$ $[=0.12+0.57t+1.005t^2+0.78t^3+0.225t^4]$ Sub $t=1$ 2.7 Attempt 2nd derivative of G_Z Attempt $G''(1)+G'(1)-G'(1))^2$ $(G''(1)=5.82)$ 1.23 Alternative methods. 3x0.5 or 2x0.6; added ; 2.7 M1M1A1 3x0.5x0.5 or 2x0.6x0.4; added; 1.23 M1M1A1 P(Z=0)=0.02 etc B1 ;E(Z)=Σzp=2.7 M1A1 E(Z²)=Σz²p=(8.52) M1;-2.7² M1 1.23A1	M1 M1dep A1 M1 M1dep A1 **[6]**	Differentiate. -ve var, M0	
	(iv)		0.335	B1ft **[1]**	Coeff t^3 from (ii)	

Question			Answer	Marks	Guidance	
7	(i)		$P(S \le s) = P(\text{at least one of } Y_1, Y_2 < s)$ $= P(\text{not both } Y_1, Y_2 > s)$ $1 - [1 - (1 - \frac{a^5}{s^5})]^2$ $P(S > s) = \frac{a^{10}}{s^{10}}$ AG CDF of $S = 1 - a^{10}s^{-10}$; and differentiate $10a^{10}s^{-11}$	M1 A1 B1;M1 A1 **[5]**	$(\frac{a^5}{s^5})^2$ cwo	
	(ii)		$E(S) = \int_a^\infty s.10a^{10}s^{-11}ds$ $\frac{10}{9}a$ $\ne a$ $\frac{9}{10}S$	M1 A1 M1 B1ft **[4]**	 Must have ans ka for E(S) Provided k>0	
	(iii)		$f(y) = 5a^5y^{-6}$ $E(Y) = \int_a^\infty y.5a^5y^{-6}dy$ $= \frac{5}{4}a$ $k = \frac{2}{5}$	B1 M1 A1 B1ft **[4]**	 1 ÷ (2×coeff of a). Must follow from an attempt at integration.	
	(iv)		Find which of T_1 and T_2 has smaller variance.	B1 **[1]**		